# Dark Energy and Dilaton Cosmology

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We studied the dilaton cosmology based on Weyl-Scaled induced gravity. The potential of dilaton field is taken as exponential form. An analytical solution of Einstein equation is found. The dilaton can be a candidate for dark energy that can explain the accelerated universe. The structure formation is also considered. We find the the evolutive equation of density perturbation, and its growth is quicker than the one in standard model which is consistent with the constraint from CMBR measurements

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## I.Introduction

The observation evidence for accelerating universe has been one of the central themes of modern cosmology for the past few years. The explanation of these observations in the framework of standard cosmology requires an exotic form of energy which violates the strong energy condition. A variational of scalar field model has been conjectured for this purpose including quintessence[1], K-essence[1] and recently tachyonic scalar fields [1].

Among these models, the important one is tachyon model. The role of tachyon field in string theory in cosmology has been widely studied. However, the model of inflation with a single tachyon field generates larger anisotropy and has troubles in describing the forming of the universe[2]. A successful extended inflation(shortened EI) can occur in Weyl-Scaled induced gravity(also was called induced gravity in the Einstein frame). During the EI epoch, the universe expands according to a power law. It has been shown that this power-law expansion is fast enough to resolve cosmological puzzles, but slow enough to percolate the false vacuum via nucleation of true vacuum bubbles[3]. The EI models also predict the primordial spectral indices of density perturbations, and the values of spectral indices is close to the observations[4]. The models predict the fluctuation as follows

$$(\frac{\delta\rho}{\rho}) \sim 10^{-5} \tag{1}$$

one can find the fluctuation $(\frac{\delta\rho}{\rho})$ well within currently observed limits[3]. A successful EI model leads to a result that we regard the dilaton field of included gravity as theoretical models of dark energy.

## II.Cosmological Dynamics In The Presence of Dilaton Field

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Let us consider the action of Jordan-Brans-Dicke theory

$$S = \int d^4x\sqrt{-\gamma}[\phi R + \omega\gamma^{\mu\nu}\frac{\partial_\mu\phi\partial_\nu\phi}{\phi} - \Lambda(\phi) + L_{fluid}(\psi)] \tag{2}$$

where the lagrangian density of cosmic fluid $L_{fluid}(\psi) = \frac{1}{2}\gamma^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - V(\psi)$, $\gamma$ is the determinant of the Jordan metric tensor $\gamma_{\mu\nu}$, $\omega$ is the dimensionless coupling parameter, $R$ is the contracted $R_{\mu\nu}$. The metric sign convention is(-,+,+,+). The quantity $\Lambda(\phi)$ is a nontrivial potential of $\phi$ field. When $\Lambda(\phi) \neq 0$ the action of Eq.(2) describes the induced gravity. The energy density of cosmic fluid $\widetilde{\rho} = \frac{1}{2}(\frac{d\psi}{d\widetilde{t}})^2 + V(\psi)$ and the pressure $\widetilde{p} = \frac{1}{2}(\frac{d\psi}{d\widetilde{t}})^2 - V(\psi)$.

However it is often useful to write the action in terms of the conformally related Einstein metric. We introduce the dilaton field $\sigma$ and conformal transformation as follows

$$\phi = \frac{1}{2}e^{\alpha\sigma} \tag{3}$$

$$\gamma_{\mu\nu} = e^{-\alpha\sigma}g_{\mu\nu} \tag{4}$$

where $\alpha^2 = \frac{\kappa^2}{2\omega+3}$. $\kappa^2 = 8\pi G$ is taken to be one for convenience.
The action(2) becomes Eq.(5) by performing the conformal transformation Eq.(3) and Eq.(4)

$$S = \int d^4x\sqrt{-\gamma}[\frac{1}{2}\tilde{R}(g_{\mu\nu}) + \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - W(\sigma) + \tilde{L}_{fluid}(\psi)] \tag{5}$$

where $\tilde{L}_{fluid}(\psi) = \frac{1}{2}g^{\mu\nu}e^{-\alpha\sigma}\partial_\mu\psi\partial_\nu\psi - e^{-2\alpha\sigma}V(\psi)$

The transformation Eq.(3) and Eq.(4) are well defined for some $\omega$ as $-\frac{3}{2} < \omega < \infty$. The conventional Einstein gravity limit occurs as $\sigma \to 0$ for an arbitrary $\omega$ or $\omega \to \infty$ with an arbitrary $\sigma$.

The nontrivial potential of the $\sigma$ field, $W(\sigma)$ can be a metric scale form of $\Lambda(\phi)$. Otherwise, one can start from Eq.(5), and define $W(\sigma)$ as an arbitrary nontrivial potential. $g_{\mu\nu}$ is the pauli metric. Cho and Damour et.al pointed out that the pauli metric can represent the massless spin-two graviton in induced gravitational theory[5]. Cho also pointed out that in the compactification of Kaluza-Klein theory, the physical metric must be identified as the pauli metric because of the the wrong sign of the kinetic energy term of the scalar field in the Jordan frame. The dilaton field appears in string theory naturally.

By varying the action Eq.(5), one can obtain the field equations of Weyl-scaled induced gravitational theory.

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{1}{3}\{[\partial_\mu\sigma\partial_\nu\sigma - \frac{1}{2}g_{\mu\nu}\partial_\rho\sigma\partial^\rho\sigma] - g_{\mu\nu}W(\sigma)$$

$$+e^{-\alpha\sigma}[\partial_\mu\psi\partial_\nu\psi - \frac{1}{2}g_{\mu\nu}\partial_\rho\psi\partial^\rho\psi] - g_{\mu\nu}e^{-2\alpha\sigma}V(\psi)\} \tag{6}$$

$$\Box\sigma = \frac{dW(\sigma)}{d\sigma} - \frac{\alpha}{2}e^{-2\alpha\sigma}g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - 2\alpha e^{-2\alpha\sigma}V(\psi) \tag{7}$$

$$\Box\psi = -\alpha g_{\mu\nu}\partial_\mu\psi\partial_\nu\sigma + e^{-\alpha\sigma}\frac{dV(\psi)}{d\psi} \tag{8}$$

The energy-momentum tensor $T_{\mu\nu}$ of cosmic fluid is

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu + pg_{\mu\nu} \tag{9}$$

where the density of energy

$$\rho = \frac{1}{2}\dot{\psi}^2 + e^{-\alpha\sigma}V(\psi) \tag{10}$$

the pressure

$$p = \frac{1}{2}\dot{\psi}^2 - e^{-\alpha\sigma}V(\psi) \tag{11}$$

$\rho$ and $p$ are related to their directly measurable counterparts by $\rho = e^{-\alpha\sigma}\tilde{\rho}, p = e^{-\alpha\sigma}\tilde{p}$. We work in R-W metric

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2) \tag{12}$$

and we consider that $\rho, p$ and $\sigma$ depend only on time. So, according to Eqs.(6)-(12), we can obtain

$$(\frac{\dot{a}}{a})^2 = \frac{1}{3}[\frac{1}{2}\dot{\sigma}^2 + W(\sigma) + e^{-\alpha\sigma}\rho] \tag{13}$$

$$\ddot{\sigma} + 3H\dot{\sigma} + \frac{dW}{d\sigma} = \frac{1}{2}\alpha e^{-\alpha\sigma}(\rho - 3p) \tag{14}$$

$$\dot{\rho} + 3H(\rho + p) = \frac{1}{2}\alpha\dot{\sigma}(\rho + 3p) \tag{15}$$

Exponential potential attract much attention because they can be derived from the effective interaction in string theory and Kaluza-Klein theory. Their roles in cosmology have also been widely investigated[6].

In this paper we assume that $W(\sigma) = Ae^{-\beta\sigma}$. From Eq.(14) we can obtain the solution in dilaton energy-dominanted epoch,

$$\sigma = \frac{2}{\beta}ln[\sqrt{\frac{A}{2(\frac{6}{\beta^2} - 1)}}|\beta|t] \tag{16}$$

For radiation $\rho_r = 3p_r$, we get $\rho_r \propto \frac{e^{\alpha\sigma}}{a^4}$ from Eq.(15). For matter with $p_m = 0$, we get $\rho_m \propto \frac{e^{\frac{1}{2}\alpha\sigma}}{a^3}$ from Eq.(15). The effective energy density of dilaton field is $\rho_\sigma = \frac{1}{2}\dot{\sigma}^2 + W(\sigma)$, and the effective pressure is $p_\sigma = \frac{1}{2}\dot{\sigma} - W(\sigma)$.

At the very large cosmological scale , the contributions from matter and radiation in Eq.(13) become negligible compared with the dilaton field. In order to see this more clearly, we take the example that when the equation of state of the dark energy(dilaton energy) is constant and $\frac{p_\sigma}{\rho_\sigma}$ must be less than $-\frac{1}{3}$, the expansion of the universe accelerates. Thus the dark energy component will evolve with $\rho_\sigma \propto a^{-3(1+\frac{p_\sigma}{\rho_\sigma})}$, which dissipates slower than radiation($e^{-\alpha\sigma}\rho_r \propto a^{-4}$) and matter($e^{-\alpha\sigma}\rho_m \propto \frac{t^{-\frac{\alpha}{\beta}}}{a^3}$). $\alpha^2 = \frac{1}{2\omega+3} < 10^{-3}$ which is constrained by present-day solar system test[4]. So at late time the dilaton energy ultimately dominates in universe, and the Eq.(13) becomes

$$(\frac{\dot{a}}{a})^2 = \frac{1}{3}[\frac{1}{2}\dot{\sigma}^2 + W(\sigma)] \tag{17}$$

From Eqs.(14) and (17), we obtain

$$a(t) = a_0(\frac{t}{t_0})^{\frac{2}{\beta^2}} \tag{18}$$

and solution of $\sigma$ is Eq.(16). After investigating stability of the solutions (16) and (18), we can find that the Eqs.(16) and (18) are stable for $\beta^2 < 2$. This is similar to $\Phi$CDM[8].
The effective energy density of dilaton is

$$\rho_\sigma = \frac{Ae^{-\beta\sigma}}{1-\frac{\beta^2}{6}} \tag{19}$$

The effective pressure is

$$p_\sigma = \frac{A(\frac{\beta^2}{3}-1)e^{-\beta\sigma}}{1-\frac{\beta^2}{6}} \tag{20}$$

From Eqs.(18) and (19), one can get

$$\frac{p_\sigma}{\rho_\sigma} = (\frac{\beta^2}{3}-1) \tag{21}$$

When $\beta^2$ is smaller than 2, Eq.(21) shows that $\rho_\sigma + 3p_\sigma < 0$ and the universe is undergoing a phase of accelerated expansion. It is clear that the $\rho_\sigma + p_\sigma$ is greater than zero from Eq.(21). When $\beta$ approximates zero, $\rho_\sigma = -p_\sigma$.

## III.The Discussion of Cosmological Structure Formation In Dilaton Cosmology

We will find that the growth of density perturbation is quicker than the one in standard model. Eq.(13) becomes

$$(\frac{\dot{a}}{a})^2 = \frac{1}{3}[\frac{1}{2}\dot{\sigma}^2 + W(\sigma) + e^{-\alpha\sigma}\rho] \tag{22}$$

The cosmic fluid can be regard as the nonrelativistic matter $p \ll \rho$. From Eqs.(22),(14)and (15), we obtain

$$\sigma = ln\sigma_0 + \frac{2}{\beta}ln[\sqrt{\frac{A}{2(\frac{6}{\beta^2}-1)}}|\beta|t] \tag{23}$$

$$a = a_e(\frac{t}{t_e})^{\frac{2}{3}-\frac{\alpha}{3\beta}} \tag{24}$$

$$\rho = \rho_e(\frac{t}{t_e})^{-2+\frac{2\alpha}{\beta}} \tag{25}$$

The radiation density equals matter density at $t_e$, correspondingly the cosmological scalar is $a_e$ and the matter density is $\rho_e$. In the early time of the matter energy-dominated epoch, $ln\sigma_0 >> \frac{2}{\beta}ln[\sqrt{\frac{A}{2(\frac{6}{\beta^2}-1)}}|\beta|t]$ and thus $W(\sigma)$ approximates to a constant $A{\sigma_0}^{-\beta}$. However the terms of $e^{-\alpha\sigma}\rho$ decreases as $t^{-2}$, so the Universe becomes

dominated by the dilaton energy little by little.
For $p = 0$ the Eq.(15) becomes

$$\dot{\rho} + 3H\rho = \frac{1}{2}\alpha\dot{\sigma}\rho \tag{26}$$

The Eq.(26) of motion of fluid can also be written as follows

$$\frac{\partial[e^{-\alpha\sigma}\rho(t)]}{\partial t} + \nabla\cdot[e^{-\alpha\sigma}\rho\vec{V}] = -\frac{1}{2}\alpha\dot{\sigma}\rho e^{-\alpha\sigma} \tag{27}$$

In standard model, the equation of motion of comic fluid is

$$\dot{\rho} + 3H\rho = 0$$

namely

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{V}) = 0 \tag{28}$$

The Eq.(27) is different from Eq.(28) due to the effect of dilaton field. when $\dot{\sigma} = 0$, Eq.(27) becomes Eq.(28)

Considering a fluctuating region which is by far smaller than the universe, we can deal with it by the following theory of fluctuation[7]. The Euler equation is

$$\frac{\partial\vec{V}}{\partial t} + (\vec{V}\cdot\nabla)\vec{V} = -\frac{1}{\rho}\nabla p + \vec{g} \tag{29}$$

where $\vec{V}$ is the speed of fluid, $\vec{g}$ is the intensity gravitational field exerted by fluid, and $\vec{g}$ satisfies the following equations

$$\nabla\times\vec{g} = 0 \tag{30}$$

$$\nabla\vec{g} = -4\pi G\rho \tag{31}$$

According to Eqs.(22)(27)(29)(30)(31), we can obtain a simple spacial homogeneous solution

$$a(t) = a_e(\frac{t}{t_e})^{\frac{2}{3}-\frac{\alpha}{3\beta}} \tag{32}$$

$$\rho(t) = \rho_e(\frac{t}{t_e})^{-2+\frac{2\alpha}{\beta}} \tag{33}$$

$$\vec{V} = \vec{r}\,[\frac{\dot{a}(t)}{a(t)}] \tag{34}$$

$$\vec{g} = -\vec{r}\,(\frac{4\pi G\rho}{3}) \tag{35}$$

Eq.(34) is a solution that satisfies the Hubble law. Now we solve the perturbation solution. One can add the zero series solution $\rho, p, \vec{V}, \vec{g}$ to the perturbation solution

$\rho_1, p_1, \vec{V_1}, \vec{g_1}$. According to Eqs.(27)and(29), one can get the first series approximate expression

$$\frac{\partial \rho_1}{\partial t} + \frac{3\dot{a}}{a}\rho + \frac{\dot{a}}{a}(\vec{r}\cdot\nabla)\rho_1 + \rho\nabla\cdot\vec{V_1} - \frac{1}{2}\alpha\dot{\sigma}\rho_1 = 0 \tag{36}$$

$$\frac{\partial \vec{V_1}}{\partial t} + \frac{\dot{a}}{a}\vec{V_1} + \frac{\dot{a}}{a}(\vec{r}\cdot\nabla)\vec{V_1} = -\frac{1}{\rho}\nabla p + \vec{g_1} \tag{37}$$

From Eq.(30) and Eq.(31), one can obtain respectively

$$\nabla\times\vec{g_1} = 0 \tag{38}$$

$$\nabla\cdot\vec{g_1} = -4\pi G\rho_1 \tag{39}$$

Assuming the perturbation is adiabatic, one knows that $p_1$ is decided by the following equation

$$p_1 = V_s^2\rho_1 \tag{40}$$

where $V_s$ is sonic speed. Because Eqs.(36)-(39) is spacial homogeneous, one expects a plane wave solution. In fact, according to the spacial dependence

$$\rho_1(\vec{r},t) = \rho_1(t)exp\{\frac{i\vec{r}\cdot\vec{q}}{a(t)}\} \tag{41}$$

and the analogous expression of $\vec{V_1}$ and $\vec{g_1}$, one can obtain these solutions(when the factor $\frac{1}{a}$ appears in the exponential, it means that the wavelength is enlarged by the expansion of universe). Substituting Eq.(41), the analogous expression of $\vec{V_1}$ and $\vec{g_1}$ into Eqs.(36)-(39), one can obtain

$$\frac{\partial \rho_1}{\partial t} + \frac{3\dot{a}}{a}\rho_1 + ia^{-1}\vec{q}\cdot\vec{V_1}\rho - \frac{1}{2}\alpha\dot{\sigma}\rho_1 = 0 \tag{42}$$

$$\dot{\vec{V_1}} + \frac{\dot{a}}{a}\vec{V_1} = -\frac{iV_s^2}{\rho a}q\rho_1 + \vec{g_1} \tag{43}$$

$$\vec{q}\times\vec{g_1} = 0 \tag{44}$$

$$i\vec{q}\cdot\vec{g_1} = -4\pi G\rho_1 a \tag{45}$$

Eqs.(44) and (45) have the obvious solution

$$\vec{g_1} = \frac{4\pi G\rho_1 a\vec{q}\vec{i}}{q^2} \tag{46}$$

For solving the equation of motion, it is convenient to separate the $\vec{V}$ into the parallel part and vertical part of $\vec{q}$

$$\vec{V_1}(t) = \vec{V_{1\perp}}(t) + i\vec{q}\varepsilon(t) \tag{47}$$

where $\vec{q}\cdot\vec{V_{1\perp}} = 0$

$$\varepsilon = \frac{-i\vec{q}\cdot\vec{V_1}}{q^2} \tag{48}$$

Introducing the relative change of density

$$\delta(t) = \frac{\rho_1(t)}{\rho(t)} \tag{49}$$

Eq.(43) is decomposed into two separate equations

$$\dot{\vec{V_{1\perp}}} + \frac{\dot{a}}{a}\vec{V_{1\perp}} = 0 \tag{50}$$

$$\dot{\varepsilon} + \frac{\dot{a}}{a}\varepsilon = (-\frac{V_s^2}{a} + \frac{4\pi G\rho a}{q^2})\delta \tag{51}$$

Taking Eq.(49) into Eq.(42), one can obtain

$$\varepsilon = \frac{\dot{\delta}a}{q^2} \tag{52}$$

Taking $\varepsilon$ and $\dot{\varepsilon}$ obtained from Eq.(52) into Eq.(51), one can obtain

$$\ddot{\delta} + \frac{2\dot{a}}{a}\dot{\delta} + (\frac{V_s^2 q^2}{a^2} - 4\pi G\rho)\delta = 0 \tag{53}$$

It was a basic equation which controlled the increase or decline of gravitational agglomeration in universe. When the energy density of radiation decreased below the rest mass density, the matter epoch began, and the above nonrelativistic perturbation theory became valid.

We consider the solution when $p = 0$. It is well known that the galaxies were exerted by the perturbation of matter density. How many times have the original perturbation increased from recombination time to present? For answering the question, we ignore $\frac{V_s^2 q^2}{a^2}$ in order to simplify the above equation, so, Eq.(53) becomes

$$\ddot{\delta} + \frac{2\dot{a}}{a}\dot{\delta} - 4\pi G\rho\delta = 0 \tag{54}$$

Substitute Eq.(32)(33) into Eq.(54), and taking $8\pi G = 1$ for convenience, one can obtain

$$\ddot{\delta} + 2(\frac{2}{3} - \frac{\alpha}{3\beta})\frac{\dot{\delta}}{t} - \frac{1}{2}\rho_e(\frac{t}{t_e})^{-2+\frac{2\alpha}{\beta}}\delta = 0 \tag{55}$$

In the standard model, we know the solution of perturbation $\delta \propto \widetilde{t}^{2/3}$. The relation between physical cosmological time and conformal time is

$$\int d\widetilde{t} = \int e^{-\frac{1}{2}\alpha\sigma}dt \tag{56}$$

namely

$$\widetilde{t} \propto t^{\frac{\beta-\alpha}{\beta}} \tag{57}$$

So

$$\delta \propto t^{\frac{2}{3}-\frac{2\alpha}{3\beta}} \tag{58}$$

Since the latter is tightly constrained from CMBR measurements, models with slow growth of perturbation can be ruled out. However, our model of dilaton cosmology is viable because the growth of density perturbation in figs.1-2 is quicker than the one in standard model. The dilaton couples weakly to ordinary matter($\alpha^2 < 10^{-3}$, which seems to argue against the existence of long-range scalars. Perhaps such a pessimistic interpretation of the limit is premature.[5]), so the difference between our model and $\Phi$CDM is small. Just like General Relativity as a cosmological attractor of induced gravity[5], the $\Phi$CDM is a cosmological attractor of our model. The dilaton-field perturbation didn't grow during the matter-dominated epoch. In dilaton-field dominated epoch, the presence of a substantial amount of dilaton-field energy density inhibits the growth of perturbations in the baryonic fluid. This is one of the reasons why the dilaton field could have come to dominate the Universe only recently if galaxies were formed by gravitational instability.

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